Noise-induced Bessel-like oscillations of Shapiro steps with the period of the ac force

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The frequency dependence of the dynamical-mode-locking phenomena in the presence of noise is studied in the one-dimensional ac driven dissipative Frenkel-Kontorova model. It was found that, besides producing the melting of Shapiro steps and decrease in the critical depinning force, noise may transfer a system by decreasing the dc threshold value into the high amplitude regime where the oscillations of the step width with the frequency of ac force will appear. Expressed as a function of the ac period, these oscillations have the Bessel-like form, the same as the well-known Bessel-like oscillations with amplitude. They may appear in any real system, irrespectively of the numbers of degrees of freedom, and strongly affect the stability of Shapiro steps, which is of great importance in the technical applications of interference effects.

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In the examinations of the interference phenomena in realistic systems, attention is always focused on the conditions that provide existence and the maximum stability of Shapiro steps. Due to possible technical applications, in recent years, many studies of the dynamical-mode-locking phenomena in the systems such as charge-density wave conductors[1–4] and systems of Josephson-junction arrays[5–10] have been particularly focused on the influence of noise. These very complex dissipative many-body systems are closely related to the dynamics of the dissipative (overdamped) Frenkel-Kontorova (FK) model.[1,11] Motivated by the great significance of the noise problem for the experiments and technical applications and the fact that until now the ac driven FK model has been studied only in the ideal case, at zero temperature[12] in this work, we will study the one-dimensional dc+ac driven overdamped FK model in the presence of noise.

The Frenkel-Kontorova model represents a chain of harmonically interacting particles subjected to a sinusoidal substrate potential.[13] It describes different commensurate or incommensurate structures that, under an external driving force, show very rich dynamical behavior. In the presence of an external dc+ac driving force, the dynamics is characterized by the appearance of the staircase macroscopic response or Shapiro steps in the response function of the system.[11] These steps are due to the interference (synchronization) or dynamical-mode locking of the internal frequency (that comes from the motion of particles over the periodic substrate potential) with the frequency of an external ac force.

We will examine how the presence of noise affects the existence and the properties of Shapiro steps, in particular, their frequency dependence since there in the standard FK model a very interesting phenomenon (frequency oscillations of Shapiro steps) has been observed in our previous work.[12] The fact that this phenomenon has been discovered in an ideal case and has not been predicted by the well-known theories about Shapiro steps raised the question of whether it is only a peculiarity of the standard FK model or if it could exist and be experimentally observable in real systems. The results have shown that besides the melting of Shapiro steps and the strong decrease in the critical depinning force, noise may transfer a system to the high amplitude regime where the oscillations of the step width with the frequency appear. This interesting phenomenon brings an insight into the theory of ac driven dynamics by revealing the analogy between the amplitude and period of the ac force. Since these oscillations may strongly affect the stability of Shapiro steps in real systems due to significance for the experiments and technical applications, some suggestions about taming these phenomena have been given.

We consider the dissipative (overdamped) dynamics of a one-dimensional chain of coupled harmonics oscillators u_l subjected in a sinusoidal substrate (pinning) potential $V(u) = \frac{K}{(2\pi)^2}[1-\cos(2\pi u)]$, where K is the pinning strength. The system is driven by the dc and ac forces $F(t)=\bar{F} + F_{ac}\cos(2\pi\nu_0 t)$. The equations of motions are

$$\dot{u}_l = u_{l+1} + u_{l-1} - 2u_l - V'(u_l) + F(t) + L_l(t), \tag{1}$$

where $l=-\frac{N}{2},\ldots,\frac{N}{2}$ and the thermal noise[9,10] satisfies $\langle L_l(t)L_l(t')\rangle = 2T\delta(t-t')$. Equation (1) has been integrated using the periodic boundary conditions for the commensurate structure with the interparticle average distance (winding number) $\omega=\langle(u_{l+1}-u_l)\rangle$ (ω is rational for the commensurate and irrational for the incommensurate structures). The commensurate structure $\omega=\frac{1}{2}$ (two particles per one potential well) has been analyzed. The time step used in the simulations was 0.001τ ($\tau=1/\nu_0$). The particles were first thermalized at zero force; then the force was increased with the step 10^{-4}, where after each step a relaxation time 100τ was used to allow the system to reach the steady state. The system size and commensurability effects have been tested, and they are important only at very high temperatures $T>1$. The response function $\bar{v}(\bar{F})$, in particular, the step width, and the critical depinning force are analyzed for different amplitudes and frequencies of the ac force at different levels of noise. At $T=0$ (without the noise), the step size can be measured precisely while in the presence of noise, the edges of the steps are rounded, and therefore, some criterion must be always made on how to measure the size of steps (which values of $\bar{F}$ determine the beginning and the end of step). We consider

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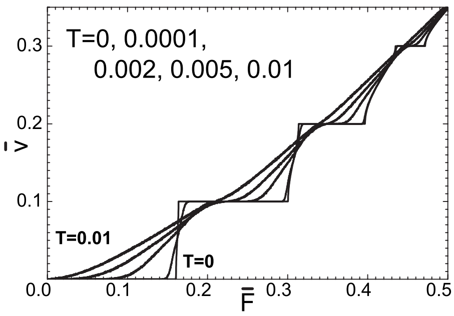

FIG. 1. Average velocity as a function of the average driving force for $\omega = \frac{1}{2}$, $K=4$, $F_{ac}=0.2$, $\nu_0=0.2$, and different values of temperature $T=0$, 0.0001, 0.002, 0.005, and 0.01.

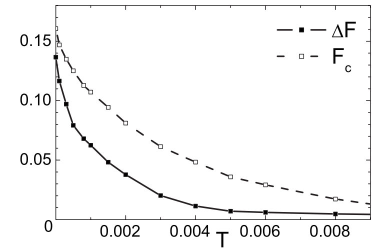

FIG. 2. The width ΔF of the first harmonic step and the critical depinning force F_c as a functions of temperature for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.2$, and $\nu_0=0.2$.

the system to be on the step if the changes in $\bar{v}(\bar{F})$ are less than 0.1% (using stronger or weaker criteria affects only quantitatively the step width but does not change qualitatively the results and the observed phenomena).

In the ac driven systems, the presence of two frequency scales can result in the appearance of the synchronization phenomena (resonance). The ac force induces additional polarization energy into the system that is different from zero (less than zero) only when the velocity reaches the resonant values $\bar{v}=\frac{i\omega+j}{m}\nu_0$ (i, j, and m are integers where $m=1$ for harmonic steps, $m>1$ for subharmonic steps, and $m=2$ for fractional or half-integer steps).[11] In the same time, the average pinning force will also be different from zero, and the system will get locked since the average pinning energy of the locked state (on the step) is lower than that of the unlocked state. As $\bar{F}$ increases, the particles will stay locked until the pinning force can cancel the increase in $\bar{F}$. The presence of noise will bring an additional contribution to the energy of particles and, therefore, strongly affect the mode locking. In Fig. 1, the response functions $\bar{v}(\bar{F})$ for the commensurate structure $\omega=\frac{1}{2}$ are presented for different values of temperature.

As T increases, Shapiro steps start to melt, becoming more and more rounded, and completely disappear; meanwhile the critical depinning force F_c decreases. At the high temperature, the pinning potential can be neglected, and the system behaves as a system of free particles. We will consider only the behavior of the harmonic steps since, in the standard FK model,[11] fractional steps can appear only for the rational noninteger values of ω. However, even at $T=0$ their size is too small, and at any temperature different from zero, they disappear. The higher order subharmonic steps can appear in the nonstandard FK model.[14]

Decrease in the step width ΔF for the first harmonic $\bar{v}=\frac{1}{1}\omega\nu_0$ and the critical depinning force F_c with the increase in temperature is presented in Fig. 2.

As we can see, with the increase in noise, the step size and the critical depinning force strongly decrease. At high T, the energy contribution from the noise will completely cancel the negative contribution from the polarization energy due to resonance, and the steps will disappear. These results are in good qualitative agreement with some previous experimental results obtained in the systems of Josephson junctions[5,6] where the same temperature dependence of the critical current and first harmonic step width have been observed.

The stability of the steps is strongly influenced by the system parameters, and the well-known Bessel-like oscillations of the step size with the ac amplitude have been examined in many works.[2] However, the presence of noise in the system can completely change the way the steps respond to the changing of system parameters. Here, we will focus on the frequency dependence (results for the amplitude dependence and disappearance of the Bessel-like oscillations of the step width due to noise will be published elsewhere).

In Fig. 3, the frequency dependence of the first harmonic step width ΔF ($\bar{v}=\frac{1}{1}\omega\nu_0$) and the critical depinning force F_c at four different values of temperature are presented.

In the FK model, at $T=0$ after initial increase, Shapiro steps gradually decrease to zero; meanwhile the critical depinning force saturates to the frequency independent threshold value for dc driven system F_{c0}. In the presence of noise, the family of the curves appears as the temperature increases, showing the strong reduction in ΔF and F_c. However, at $T=0.004$, we observed the unusual nonmonotonic increase at low frequencies similar to the one that has been observed in the standard FK model ($T=0$) in the high amplitude regime.[12] In the inset of Fig. 3(a) that shows enlarged curve ΔF in the region of low frequencies, we can clearly see that the step width oscillates with the frequency.

The physical origin of these low-frequency oscillations can be understood if the step width is analyzed as a function of period $\frac{1}{\nu_0}$. In Fig. 4, ΔF as a function of $\frac{1}{\nu_0}$ is presented for two different values of temperature. The results clearly show a great difference between two curves, where at $T=0.004$ the oscillatory dependence appears.

As in the case of the Bessel-like oscillations of the step width with the ac amplitude,[2] these low-frequency oscillations are the result of the simultaneous competition and contributions of the dc and ac components of the force $F(t)$ to the pinning energy.[12] Due to the presence of noise, the dc threshold F_{c0} decreases, and it may become smaller than the ac amplitude F_{ac}. Therefore, the system may change from the low amplitude regime $F_{ac} \leq F_{c0}$ to the high one $F_{ac} > F_{c0}$. For the case in Figs. 3 and 4, the system is driven by the ac force with amplitude $F_{ac}=0.2$, and at $T=0.001$, $F_{ac} < F_{c0}$ since $F_{c0}=0.2544$. However at $T=0.004$, the dc threshold decreases to $F_{c0}=0.1567$ and the system is now in regime $F_{ac} > F_{c0}$. When $\frac{F_{ac}}{F_{c0}} > 1$, the ac contribution, which is responsible for the appearance of these oscillations, will dominate

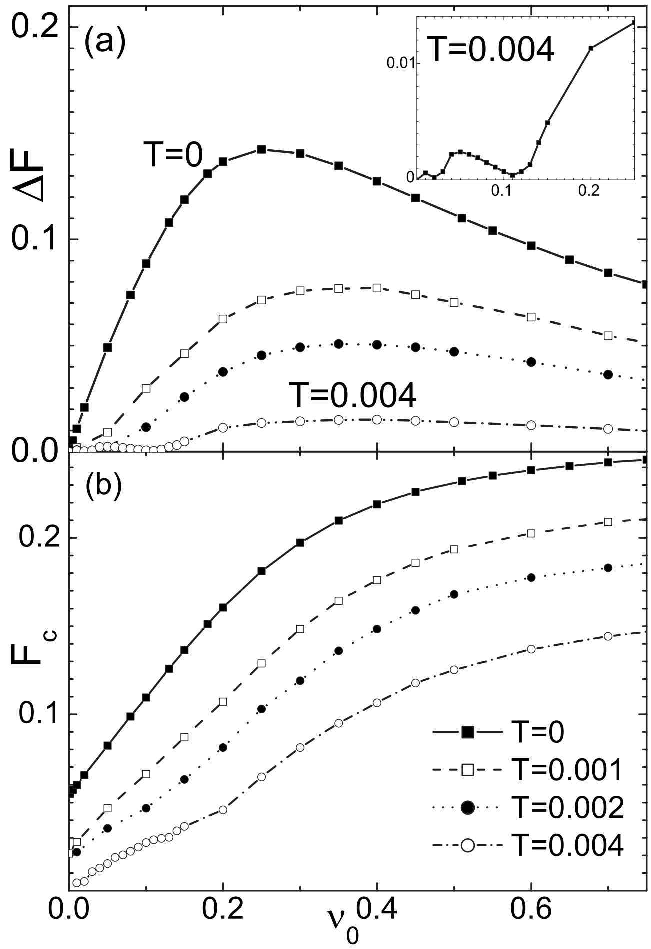

FIG. 3. (a) The width ΔF of the first harmonic step, and (b) the critical depinning force F_c as a function of frequency for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.2$, and $T=0$, 0.001, 0.002, and 0.004. The inset shows the enlarged ΔF curve for $T=0.004$.

in the pinning energy. The oscillations appear due to the backward and forward motions of particles induced by the ac force, where not only the ac amplitude but also the period (frequency) determines how much this motion is retarded. For the values of the period that correspond to the first maximum, particles will spend most of the time pinned and then hop to the next well, while for the values at the second maximum, particles will jump one site back and two forward. As the period increases, the particles will hop between the wells that are more and more distant while staying less and less time pinned and, consequently, the step width will decrease. Disappearance of this oscillatory behavior at the high frequencies (low period), as we can see in the inset of Fig. 3, appears at the point where the frequency reaches the value for which the dc contribution will cancel the ac contribution (Shapiro steps could be also produced by changing ν_0 and keeping $\bar{F}$ constant).

If the ratio $\frac{F_{ac}}{F_{c0}}$ increases, the oscillations will spread more toward the higher frequencies while the maxima will increase, and for $\frac{F_{ac}}{F_{c0}} \gg 1$, the oscillatory behavior will dominate. In Fig. 5, the step width ΔF and the critical depinning force F_c are presented both as a function of ν_0 in Fig. 5(a),

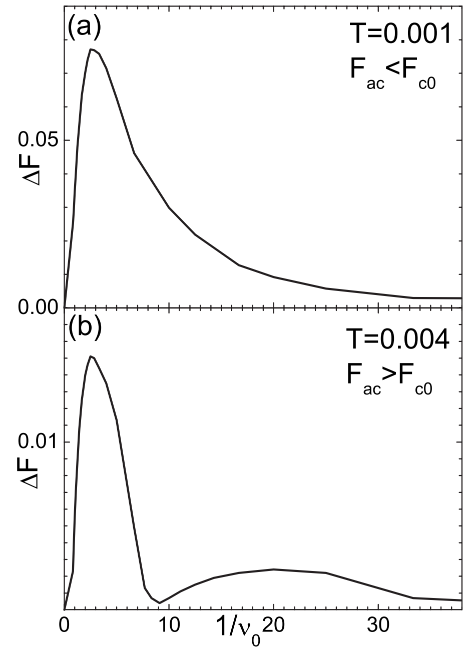

FIG. 4. The width of the first harmonic step ΔF as a function of $(\frac{1}{\nu_0})$ for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.2$, and two different values of temperature: (a) $T=0.001$ and (b) 0.004.

and as a function of $\frac{1}{\nu_0}$ in Fig. 5(b) for four different values of temperature.

The system is now driven by the ac force $F_{ac}=0.5$ ($F_{ac} > F_{c0}, F_{c0}=0.2544$), and at $T=0$, the system is already in the high amplitude regime. If we compare two curves at the same temperature $T=0.004$: the one in the inset of Fig. 3 for $F_{ac}=0.2$ and the other in Fig. 5(a) for $F_{ac}=0.5$, we can clearly see that oscillations are moved to the higher frequencies with much higher and more pronounced maxima as F_{ac} increases [in Fig. 3, steps are unstable for $\nu_0<0.11$, while in Fig. 5(a), they are unstable for $\nu_0<0.2$]. Similar effects can be seen in Fig. 5(b), where at the same temperature, the maxima of ΔF for $F_{ac}=0.5$ are much higher compared with the results in Fig. 4 for $F_{ac}=0.2$. At $T=0$, the oscillations in Fig. 5(b) have Bessel-like form where maxima of ΔF curves correspond to the minima of F_c curves. These results in Figs. 4 and 5(b) (compared with the results for amplitude dependence[2]) clearly reveal an analogy between the amplitude and the period of the ac force. Increase in the period has

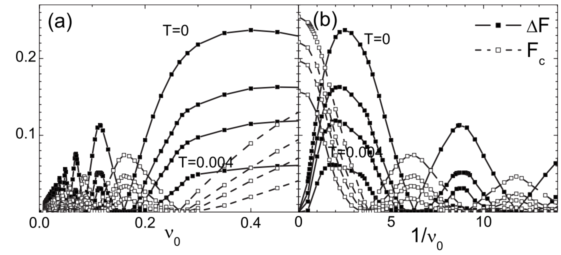

FIG. 5. The width ΔF of the first harmonic step and the critical depinning force F_c as a function of (a) ν_0 and (b) $(\frac{1}{\nu_0})$, for $\omega=\frac{1}{2}$, $K=4$, $F_{ac}=0.5$, and $T=0$; 0.001, 0.002, and 0.004 from the top to the bottom, respectively.

a similar effect on the back and forward motions of particles and, therefore, has a similar effect on the step size as the increase in amplitude. Displacement between more distant sites will appear only if the amplitude is high enough or the period is long enough. As noise (T) increases, due to the increase in energy, the particles will move between more and more distant sites, and stay less pinned. Therefore, the step size will decrease while the Bessel-like form will disappear. When $\nu_0 \to \infty$ or at $\frac{1}{\nu_0}=0$, in Figs. 5(a) and 5(b), respectively, F_c goes to the dc threshold value F_{c0} that decreases with the increase in noise.

Frequency dependence of Shapiro steps has been a matter of many controversies. In the charge-density wave systems, according to the classical approach,[4] the step width and the critical depinning force should, after initial increase, decrease to zero at the high frequencies. In contrast, according to the simple single coordinate model motivated by the tunneling theory,[2] the maximum step width and the magnitude of the fundamental component of the effective pinning force are independent of frequency at the high frequencies. In the systems with Josephson-junction arrays, according to the single junction model,[15] the widths of harmonic steps remain frequency independent at the high frequencies. On the other side, in the systems with many degrees of freedom,[16] disappearance of steps at the high frequencies has been observed (single junction models do not work well if the system is disordered). The FK model that we have considered is an overdamped classical many-body model, and as in other systems with many degrees of freedom, the steps will remain strongly frequency dependent and disappear at the high frequencies. In order to examine whether these frequency oscillations appear in the single degree of freedom systems, we have also analyzed the commensurate structure with the winding number $\omega=1$, for which the FK model reduces to the single-particle model at $T=0$.[11] As it was shown in our previous work at $T=0$,[12] and any temperature for which Shapiro steps exist, we observe these oscillations in any commensurate structure always when $\frac{F_{ac}}{F_{c0}}>1$.

The presented results have shown that the oscillations of Shapiro steps with the period not only exist in realistic systems but the environmental effects such as noise could even induce them and, by that, significantly influence the stability of the steps. Oscillations of Shapiro steps with frequency (period) are universal and will appear always when $\frac{F_{ac}}{F_{c0}}>1$, as a result of the physical process (back-forward motion) that is in the core of every ac driven dynamics, in any system, and irrespectively of the number of the degrees of freedom.

In real systems, environmental and all other factors (impurities, deformations, and imperfections) that have strong impact on the dc threshold F_{c0} may transfer a system from low $\frac{F_{ac}}{F_{c0}} \leq 1$ to the high amplitude regime $\frac{F_{ac}}{F_{c0}}>1$ (or vice versa), and by that, induce (or suppress) oscillations of Shapiro steps with frequency (period). In Fig. 6, it is shown how for the applied ac amplitude with the constant value $F_{ac}=0.2$, the ratio $\frac{F_{ac}}{F_{c0}}<1$ at $T=0$ becomes $\frac{F_{ac}}{F_{c0}}>1$ due to reduction in F_{c0} in the presence of noise.

This may create a great problem in the experiments or technical applications. It is well known that the steps are less

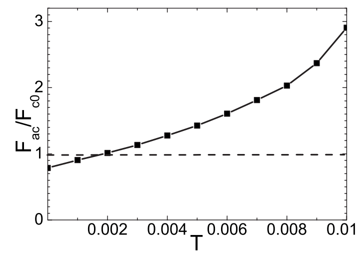

FIG. 6. The ratio F_{ac}/F_{c0} as a function of temperature for $\omega=\frac{1}{2}$, $K=4$, and $F_{ac}=0.2$.

rounded and better defined if the ac amplitude increases. However, application of the high value F_{ac} in order to tackle rounding of steps due to noise while simultaneously F_{c0} is getting even reduced could have completely contra effect if the ratio reaches the value where $\frac{F_{ac}}{F_{c0}} \gg 1$; in which case, the oscillations of Shapiro steps will become pronounced and spread even to the high frequencies. Therefore, in experiments or technical applications, all factors that may significantly change the ratio $\frac{F_{ac}}{F_{c0}}$ must be taken into account, and the parameter of the system should be adjusted in the way that $\frac{F_{ac}}{F_{c0}}$ is either smaller or around one; otherwise, the instability region with oscillations will spread to higher frequencies.

The Bessel-like oscillations with amplitude have been experimentally observed in many experiments performed in charge-density wave systems and the system of Josephson-junction arrays.[2,6,7] Since the period plays the same role as the amplitude in the ac driven dynamics, these Bessel-like oscillations with period should be then also experimentally observable in the same experiments where the results for amplitude dependence have been obtained ($\frac{F_{ac}}{F_{c0}}>1$ is a necessary condition). Charge-density wave systems[2] are particularly good candidate since the physics behind the interference phenomena in these systems is similar to the above presented in the FK model. Since there have been a relatively small number of experimental studies of frequency dependence and no studies of the period dependence that are known to us, we hope that these results will stimulate new experiments.

The presented results could be important for all areas of science, particularly for studies of the charge-density waves in solids, and the Josephson-junction arrays that are motivated by the fabrication of synchronization and superconducting devices.[1,11] Any application of interference phenomena and the building of Shapiro step devices[8] require a theoretical guideline for the observation of Shapiro steps, where, in the understanding of the environmental effects, the most important one is certainly the noise effect. These oscillations of Shapiro steps with frequency (period) may appear in any real system and strongly affect their stability, which is crucial in any technical application of interference effects.

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